a-g Integrated Math I

Course Description:

This is the first course in a 3 course program that includes Integrated Math I, II, and III. The course integrates topics from Algebra I, Geometry, Algebra 2, and Probability and Statistics. In this course, students will learn to think mathematically through collaboration, collection of data, experimentation, and conjectures. They will use critical thinking strategies to develop a deep understanding of mathematics. The course will help students discover that math makes sense. Students will make and test conjectures to justify conclusions, they will learn how to use mathematical models to represent real-world data, they will be able to provide clear and concise answers, and they will demonstrate computational and symbolic fluency.

Course Purpose:

Mathematics I is the first course in the series of higher level math courses including Mathematics II and Mathematics III. Upon completion of the series, students will be prepared for a course in Pre-Calculus. The purpose of Mathematics I is to extend the mathematics learned in the middle grades and lay the foundation for the mathematics of the higher level math courses. Conceptual understanding and procedural fluency are emphasized in the conceptual categories of Number and Quantity, Algebra, Functions, Geometry, and Statistics and Probability. Students will extend their understanding of numerical manipulation to algebraic manipulation, synthesize their understanding of function, deepen their understanding of linear relationships, apply linear models to data, prove geometric relationships involving congruence, and use the Pythagorean Theorem on the coordinate plane to prove geometric relationships.

Textbook:

Mathematics I Volume 1 and 2, Pearson Education, 2014, Charles, Randall I., et.al. Course Outline Unit 1--Solving Equations and Inequalities

This unit connects and extends the students' knowledge of variables and expressions to solving equations, inequalities, and proportions. Students use equations to describe, explain, and predict various real-world relationships. Students will solve multi-step linear and absolute value equations and inequalities with variables on both sides.

Students will:

- know and use the Distributive Property to simplify and combine expressions
- develop pathways to solving multi-step equations and inequalities
- solve equations and inequalities with variables on both sides
- use equations to model real-world situations and to solve problems
- convert measurements into different units of measure and use appropriate units in expressing answers to problems

- develop and use the Cross Products Property as a shortcut to solving proportions
- write and solve compound inequalities
- write and solve an equivalent pair of linear equations or inequalities to represent absolute value equations and inequalities

Assignments:

- Homework (text chapter 1)
- Quizzes
- Activity Labs: Unit Analysis, Accuracy and Measurement
- Performance Task: Planning a Fundraiser
- Unit Test

Unit 2--An Introduction to Functions

This unit introduces the topic of functions. Students will learn that a function is a relationship between variables in which each value of the input variable is associated with a unique value of the output variable. Functions are represented in a variety of ways, such as graphs, tables, equations, or words. Functions can be used to model real-world situations, which can then be used to make estimates or predictions about future occurrences.

Students will:

- use graphs, tables and verbal descriptions of data
- sketch graphs that represents the changing relationship between two quantities (eg. time and distance)
- construct equations from tables of data relating two quantities with a linear relationship
- interpret and construct tables and equations for nonlinear data
- graph data from functions by constructing a table of values
- write and solve equations to model real-world problems
- understand domain and range and use these terms to describe quantities in a function
- use the vertical line test to apply the definition of a function
- look for patterns in arithmetic sequences and determine a rule for the sequence

Assignments:

- Homework (text chapter 2)
- Quizzes
- Performance Task: Comparing the Growth of Two Blogs
- Unit Test

Unit 3--Linear Functions

This unit extends the learning of linear functions to more in-depth analysis of the slope of a line and the equation of a line.

Students will:

- find slope using a formula
- find slope using a graph
- analyze various slopes and describe their meaning

- write and graph direct variations and determine relationships between equations, graphs, and tables
- apply their knowledge of slopes to writing and graphing linear equations in slope-intercept form
- graph a linear equation as well as determine a linear equation from a graph using point-slope form
- correlate verbal descriptions to the standard form of a linear equation
- determine whether two lines are parallel, perpendicular or neither
- use the definitions of parallel and perpendicular lines to write equations and solve problems
- graph absolute value functions

Assignments:

- Homework (text chapter 3)
- Quizzes
- Performance Task: Analyzing Advertising Revenue
- Unit Test

Unit 4--Systems of Equations and Inequalities

This unit connects and extends the concepts associated with equations and inequalities to systems of equations and inequalities. Students will learn how to solve systems of equations by graphing, substitution and elimination. They will be able to write systems to solve real-world problems and interpret the solutions in the context of the problem.

Students will:

- graph systems of equations and find the point of intersection, if it exists
- solve systems of equations by substitution
- solve linear systems using the elimination method and identify relationships between the substitution and elimination methods
- write and solve systems to equations to find break-even points and solve mixture problems
- graph a linear inequality and identify the solution as the set of all points on one side of a boundary line
- solve systems of linear inequalities and identify the solution as the region where the graphs of individual inequalities overlap

Assignments:

- Homework (text chapter 4)
- Quizzes
- Performance Task: Planning an Exercise Program
- Unit Test
- Project: Let's Dance

Unit 5--Exponential and Radical Functions

This unit expands the students' understanding of skills related to exponential expressions. Students will learn to represent numbers using negative exponents and identify the characteristics of exponential growth and decay functions. They will also extend their knowledge of graphing functions to radical and piecewise functions.

Students will:

- know and use the properties of zero and negative exponents to find the values of expressions
- produce equations, graphs, tables and verbal explanations of exponential functions and identify relationships between the different representations
- model compound interest, radioactive decay, and other real-world situations
- analyze real-world data, using average rates of change, and determine whether the situation is best modeled by a linear or exponential function
- solve exponential equations by writing each side of the equation with the same base
- model geometric sequences with exponential functions
- add, subtract, multiply, and divide functions
- use the multiplication and division properties of square roots to simplify radicals
- graph square root and cube root functions by transforming parent functions
- graph piecewise functions

Assignments:

- Homework (text chapter 5)
- Quizzes
- Performance Task: Analyzing Sales Data
- Unit Test

Unit 6--Data Analysis

This unit introduces students to the topics of data analysis and teaches them to use data to help make decisions or make predictions.

Students will:

- calculate and compare measures of central tendency, i.e., mean, median, and mode of a data set
- calculate measures of dispersion, i.e., range, mean average deviation, standard deviation
- organize data in displays such as frequency tables, histograms, scatter plots, and box-andwhisker plots, line plots, stem-and-leaf
- write an equation of a trend line and a line of best fit for a scatterplot
- make predictions based on trend lines
- use two-way frequency tables to summarize, interpret, and analyze trends in data

Assignments:

- Homework (text chapter 6)
- Quizzes
- Performance Task: Choosing a Location for a Tournament
- Unit Test

Unit 7--Tools of Geometry

This unit introduces student to various topics in the study of geometry including visualization, reasoning and proof, and measurement. Students learn to visualize three-dimensional figures from two-dimensional drawings such as nets, isometric drawings, and orthographic drawings. They will

also explore the undefined terms of geometry--points. lines, and planes--and then build on these terms to explore the attributes of segments, rays, and angles. Basic postulates are introduced and later built upon to reason about lengths of segments and angles.

Students will:

- identify the net of a 3-dimensional figure and learn to make isometric and orthographic drawings
- make explicit use of the terms relating to points, lines, planes, segments, and rays
- use the Ruler and Segment Addition Postulates to find, compare and reason about the lengths of line segments
- use the Protractor and Angle Addition Postulates to find, compare and reason about the measures of angles
- explicitly define and use terms for special pairs, including adjacent, vertical, linear pair, complementary, and supplementary angles
- use the midpoint and distance formulas and apply them to line segments on a coordinate plane

Assignments:

- Homework (text chapter 7)
- Quizzes
- Performance Task: Solving a Riddle Unit Test
- Project: Illustrated Dictionary

Unit 8- Transformations

In this unit students learn how to change a figure without changing its size and shape by performing the rigid- motion transformations of translations, reflections, and rotations. Students will describe these transformations on and off of the coordinate plane. They will also learn to use compositions of these transformations.

Students will:

- define transformations and their properties
- distinguish between rotations, reflections, and translations
- identify line symmetry, rotational symmetry and point symmetry using reflection and rotations
- perform glide reflections

Assignments:

- Homework (text chapter 8)
- Quizzes
- Performance Task: Programming a Video Game Unit Test

Unit 9-- Connecting Algebra and Geometry

This unit introduces various topics in the study of geometry and its connection to algebra. Area and perimeter of polygons, circles, and irregular shapes will be explored. Students build on their

knowledge of a rectangle's area and develop area formulas for triangles and special quadrilaterals-kites, rhombuses, parallelograms, and trapezoids.

Students will:

- find perimeter, circumference and area of rectangles, triangles, pentagons and circles in the coordinate plane
- find the area of rhombuses, kites, and trapezoids
- use area formulas to find missing dimensions, such as base or height, of a figure
- solve real-life problems involving area
- use proper units when expressing area and perimeter
- use slope, distance, and midpoint formulas to prove geometric relationships for figures on the coordinate
- plane, eg. use the distance formula to show that a triangle is isosceles

Assignments:

- Homework (text chapter 9)
- Quizzes
- Performance Task: Finding the Area of a Plot of Land
- Unit Test

Unit 10-- Reasoning and Proof

This unit introduces topics related to reasoning. Students will learn inductive and deductive reasoning. Students will become more mathematically proficient by finding patterns and using repeated reasoning to make predictions. Students learn to reason and construct arguments using laws of deductive reasoning. In writing proofs, students formalize their ability to construct viable arguments supported by previously established definitions, postulates and theorems.

Students will:

- perform basic constructions using a straightedge and compass
- recognize patterns in numeric and geometric sequences
- make and test conjectures
- find counterexamples for false conjectures
- use inductive reasoning to solve problems
- complete truth tables for conditional statements
- write converses of conditional statements
- write inverses of conditional statements
- write contrapositives of conditional statements
- write biconditional statements
- use algebraic properties to justify the steps when solving an equation
- prove geometric statement with a logical argument, justifying each step

Assignments:

- Homework (text chapter 10)
- Quizzes
- Performance Task: Analyzing a Calendar Pattern

• Unit Test

Unit 11-- Proving Theorems About Lines and Angles

This unit expands on students' understanding of developing viable arguments and to prove statements about parallel and perpendicular lines and angle pairs formed by intersecting lines.

Students will:

- use postulates and theorems to explore lines in a plane
- recognize angle pair relationships when two or more lines are intersected by a transversal
- prove lines parallel
- discover and use the Triangle-Sum Theorem
- construct parallel and perpendicular lines

Assignments:

- Homework (text chapter 11)
- Quizzes
- Performance Task: Planning the Paths for a Park
- Unit Test
- Project: Geometric Constructions

Unit 12-- Congruent Triangles

This unit continues to build on students' understanding of constructing viable arguments to prove geometric statements. The unit focuses on proving triangles congruent.

Students will:

- prove triangles congruent by SSS, SAS, ASA, and AAS
- use corresponding parts of congruent triangles
- prove statements about isosceles and equilateral triangles prove right triangles congruent using the HL theorem

Assignments:

- Homework (text chapter 12)
- Quizzes
- Performance Task: Applying Indirect Measurement
- Unit Test

Key Assignments

Each unit will include one or more quizzes and an end-of-unit test to assess students' understanding of the concepts. A performance task will be included in the unit test. A practice performance task will be completed in class prior to the test.

Homework will be assigned from the text on a daily basis. Homework involves a variety practice

exercises to develop the students procedural fluency and engage in problem-solving applications. Students engage independently with the concepts and can access online tutorials for concept review and re-teaching, as needed. The variety of exercises address the eight Standards of Mathematical Practice.

A "Problem of the Week" will be assigned each week. Students will work on the problem independently, in pairs, or in groups. Students will be required to write an explanation of the solution to the problem, and at times discuss the solution with others or present their solution to the class. The purpose of this type of assignment is to give students more time to engage in problem-solving and not be rushed to arrive at a solution. Students will have time to construct viable arguments and critique the reasoning of others in regard to the solutions presented.

Class activities involve opportunities for students to work with other students on a hands-on activity, investigation, or extension lesson related to a topic in the current unit of study. Students will engage in at least one of the eight Standards of Mathematical Practice in each of these lessons. Brief descriptions of the activities are listed below.

Several projects will require the students to apply what they have learned to a longer term assignment. Some projects will require presentation in the form of a written report or Powerpoint presentation.

Unit 1--Solving Equations and Inequalities Class activities:

- use algebra tiles to illustrate the Distributive Property
- model and solve equations using algebra tiles
- Unit Analysis: In this extension lesson, students work through problems that can be solved with unit

analysis. They will work in pairs to complete two guided-question activities. In the final step of each activity, students must answer the following questions: 1) Are the units of your solution what you expected? Explain. 2) Does your answer make sense? Explain.

• Accuracy and Measurement: Students work in pairs to complete a guided activity involving accuracy in measuring a bag of sand.

Key Questions (to be answered as homework or classwork):

- What properties are used to solve equations?
- What are some methods that can be used to solve equations containing fractions? Decimals?
- What is an identity?
- How can you tell how many solutions an equation has?
- How is the process of rewriting literal equations similar to the process of solving equations in one variable? How is it different?
- How can you convert units of measure?
- How can the Cross Products Property be used to solve a proportion?
- How can you tell when an inequality has no solution?
- What is the difference between compound inequalities containing the words and /or?
- How many solutions do you expect when you solve an absolute value equation?
- How can you rewrite an absolute value inequality as a compound inequality?

Unit 2--An Introduction to Functions Class activities:

- generate table of values relating two quantities both linear and nonlinear; graph the ordered pairs; describe the relationship in words and as a function rule
 - Celsius vs. Fahrenheit
 - Inches vs. Centimeters
- Pattern Block Functions: In this investigation activity, students will explore, identify, and describe the relationship between perimeters and the number of tiles in a pattern block sequence. They will graph the perimeter as a function of the number of tiles for triangle, square, trapezoid and hexagon pattern blocks. They will write a rule for each relationship.

Key Questions:

- How do tables and graphs show the relationship between two quantities?
- How can you tell whether a relationship in a table is a function?
- How can a graph tell you if a function is linear or nonlinear?
- How can you graph a function rule?
- How can you decide if a real-world function is continuous or discrete?
- When is a relation not a function?
- How can you identify an arithmetic sequence?
- What information do you need to write a rule for an arithmetic sequence?
- What is the difference between an explicit rule and a recursive rule of an arithmetic sequence?

Unit 3--Linear Functions Class activities:

- Nickels and Dimes: Students will construct a table, draw a graph, and write an algebraic rule for the number of nickels and dimes that add to exactly one dollar. They will also apply the concepts of domain and range to this real-world situation.
- Functions in Circles: Students will deepen their understanding of diameters and circumferences of circles by using manipulatives, accurately measure diameter circumferences of containers, construct an input/output table of the data, graph the data, and write a rule for the function in sentence and standard algebraic form and recognize that the table, graph, and rule assign a value to pi and that the slope equals pi.

Key Questions:

- What is the slope of a linear function?
- How do you know if an equation represents a direct variation?
- How can you use slope to graph a linear function?
- How does the equation of a line help you make a graph?
- How can you use the intercepts to help you graph a line?
- How can you tell if a line is horizontal, vertical or neither by looking at its equation?
- How can you determine whether two lines are parallel, perpendicular, or neither?

Unit 4--Systems of Equations and Inequalities Class activities:

• numerous scenarios will be graphed using systems of equations to solve problems Project:

Let's Dance: Students will work through a series of activities related to planning a student dinner dance. They will use systems of equations and inequalities to analyze costs and make decisions. They will write a report or prepare a presentation detailing the choice of band, cost of a catering service, and ticket price recommendation.

Key Questions:

- How does graphing help you find the solution to a system of equations?
- How can you tell if a system of equations has one solution, no solution, or infinitely many solutions?
- When is the substitution method a better method than graphing for solving a system of linear equations?
- How can you solve a system of equations using elimination?
- How is a linear inequality in two variables like a linear equation in two variables? How are they different?
- How is finding the solution of a system of inequalities different from finding the solution of a system of equations? How is it the same?

Unit 5--Exponential and Radical Functions

Class activities:

- numerous scenarios involving exponential growth and decay will be graphed
- graph real-world step functions Key Questions:
- What are the differences between a linear function and an exponential function?
- How can you decide if a table represents a linear or an exponential function?
- What is a real-world situation that can be modeled by an exponential function?
- How can you determine if an equation represents exponential growth, exponential decay, or neither?
- What are some ways to solve an exponential equation?
- What is a geometric sequence?
- What do you need to write recursive and explicit formulas for a geometric sequence?
- How can you tell whether a radical expression is in simplified form?
- What is a real-world function that can be modeled by a piecewise function?

Unit 6--Data Analysis Class activities:

- make frequency tables and histograms to display data
- Close to Ideal: Students will note that Leonardo DaVinci believed that the ideal person's height was the same as his or her arm span, take the required measurements of themselves, use stem-and-leaf plots, box-and-whisker plots, scatterplots, line plots, and ratios to gain an understanding of central tendency, spread, and correlation of the data, and conclude how ideal they think they are.

Key Questions:

- How can you use a frequency table to help make a histogram?
- What are some ways to organize and display data?

- What is used to describe the center of a data set?
- Which measure of central tendency is most affected by an outlier?
- What is used to describe the spread of a data set?
- How do you draw an accurate trend line on a scatterplot?

Unit 7--Tools of Geometry Class activities:

- cut out and fold nets and identify the corresponding prisms and pyramids
- find all of the possible nets with only square faces that can be designed for a cubical box
- design a net that will cover a 1x2x3 rectangular solid and be cut from a reactangle with the smallest area
- build structures with cubes and them represent them with orthogonal and isometric drawings; trade drawings with a partner and duplicate the building represented by the drawings
- use protractors to measure angles

Project:

Illustrated Dictionary: Students will make an illustrated dictionary by finding photos (or taking their own) of real- world objects illustrating geometric terms, e.g., angle, circle, perpendicular lines, parallel lines, adjacent angles, etc. Students will use presentation software to present their project to the class.

Key Questions:

- How can you represent a three-dimensional figure with a two-dimensional drawing?
- What are the building blocks of geometry?
- How can you describe the attributes of a segment or angle?
- How do you know if two segments are congruent?
- How can you use a protractor to measure an angle?
- What conditions must be met for two angles to be adjacent?

Unit 8--Transformations Class activities:

- use tracing paper to translate, reflect and rotate a figure on the coordinate plane
- use paper folding to discover the relationship between a figure and its reflected image
- use a compass to draw a rotation image of a figure at various angles of rotation
- Capture the Flag: In this hands-on game, students will review the meaning of translations, reflections, and rotations on a coordinate plane. Students will translate points, reflect points across the x- and y- axes, rotate points about the origin, and use transformations in sequence to move a marker from the origin to various flag targets (i.e., ordered pairs) in the coordinate plane.

Key Questions:

- What types of transformations preserve angle measures and distance?
- How is a reflection line related to the segment joining a point and its image?
- Compare rotating a figure about a point to reflecting the figure across a line. How are the

transformations alike? How are they different?

• Do all regular polygons have line symmetry? Rotational symmetry? Point symmetry? Explain

Unit 9--Connecting Algebra and Geometry Class activities:

- graph polygons on the coordinate plane; measure or calculate the perimeter and area
- Geoboard Formulas: Parallelograms and Trapezoids--Students will make all possible parallelograms and trapezoids on a geoboard. They will measure perimeter and area by counting and develop generalized formulas from their measuring experiences.

Project:

Do-It-Yourself (DIY) Backyard Makeover: Students will design a backyard, calculate the areas and perimeters of each region in the yard, and use those calculations to determine the cost of materials for a DIY home improvement project.

Key Questions:

- How can you show that a parallelogram and a rectangle with the same bases and heights have the same area?
- How is the area of a triangle related to the area of a parallelogram?
- How can you find the area of an irregular figure?
- What information do you need to find the area of a trapezoid?
- Can you find the area of a rhombus if you only know the lengths of its sides? Explain.
- Do you need to know the lengths of the sides of a kite to find its area? Explain.
- How can you use slope to get information about the sides of a figure?
- How do you classify triangles in the coordinate plane?
- How do you classify quadrilaterals in the coordinate plane?

Unit 10 -- Reasoning and Proof Class Activities

- construct perpendicular bisectors and angle bisectors using a compass and straightedge and paper folding
- analyze advertisements for if-then conditional statements, either explicitly stated or implied; identify the converse of the statements and analyze the truth values of the statements and their converses
- complete logic puzzles Key Questions:
- What does it mean to construct a figure? How is this different from sketching or using a ruler or protractor?
- What is a conditional statement?
- How do you form the converse, inverse, and contra-positive of a conditional statement?
- What can be used as reasons in a proof?

Unit 11--Proving Theorems About Lines and Angles Class activities:

• discover the triangle angle-sum theorem by measuring angles of several triangles and finding the sums of the angles

Project:

Geometric Constructions: Students will make a portfolio of formal geometric constructions. Constructions will include copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines; constructing the perpendicular bisector of a line segment; constructing a line parallel to a given line through a point not on the line; constructing an equilateral triangle, a square, and a regular hexagon inscribed in a circle. Various construction techniques will be used, such as compass and straightedge, paper- folding, and geometry software. Students will use these construction techniques to design a logo using an inscribed regular polygon as its base.

Key Questions:

- How do you prove that two lines are parallel?
- How do you know when to use the Alternate Interior Angle Theorem and when to use the Converse of the Alternate Interior Angles Theorem?
- How would you prove the Triangle Angle-Sum Theorem?
- How does the Triangle Exterior Angle Theorem make sense based on the Triangle Angle-Sum Theorem?
- How can you use congruent angles to construct parallel lines?

Unit 12-- Congruent Triangles Class activities:

- work in partners to write a two-column proof involving triangle congruence
- use paper folding to make conjectures about the base angles of an isosceles triangle
- Perform compositions of rigid motion transformations on the coordinate plane

Key Questions:

- How do you identify corresponding parts of congruent triangles?
- How do you show that two triangles are congruent?
- How are the SSS Postulate and the SAS Postulate alike and how are they different?
- How are the ASA Postulate and the SAS Postulate alike and how are they different?
- How can the ASA Postulate be used to prove the AAS Theorem?
- What relationships exists among the sides and angles of an isosceles triangle?
- How are the HL Theorem and the SAS Postulate alike and how are they different?

Instructional Methods and Strategies

Guided Instruction (5-part lesson structure): Each lesson in a unit follows a five part lesson plan-interactive learning, guided instruction, lesson check, practice, assess and remediate-- to convey the mathematical content.

- Interactive learning-- Each unit is opened with a problem for the students to solve. Students work individually or in small groups to propose a solution plan and a solution. This provides a real-world setting for the mathematical concepts that will be presented in the unit.
- Guided instruction--Students are guided through problems and exercises that focus on key mathematical concepts
- Lesson check --Guided practice exercises are completed in the student worktext. The exercises include both procedural and short answer questions to check the students' understanding of mathematical structure and meaning.

- Practice-- Independent practice exercises include procedural, problem-solving and challenge exercises.
- Assess and remediate--Brief quizzes at the end of each lesson provide opportunities for differentiated instruction based on quiz results. Students can access online resources for reteaching, if necessary.

Guided discovery through questioning and/or hands-on investigations: As students engage in realworld activities, they not only find the mathematics interesting and relevant, but they can also see patterns and structure that lead to generalizations, such as properties, formulas, or theorems.

Partner and group discussion: Students are given opportunities during lessons and activities to verbalize their understanding of the concepts. This reinforces their understanding of the concepts and strengthens their reasoning skills.

Projects: Projects will require students to use various tools and technology appropriately. These tools include compass and straightedge, calculators, internet research, presentation software (e.g., Powerpoint), geometry software, graph paper, algebra tiles

Interactive Student Notebooks: Students will keep an interactive notebook in which they will record answers to key questions during class investigations, solution pathways to the Problem of the Week, and daily notes that are additional to those in the student worktext.

Class activities : Each unit involves activities that include real-world experiences, oral and written communication, pictorial or graphic representation, and critical thinking. These activities support all of the eight Standards of Mathematical Practice.

Assessments

Quizzes: Brief quizzes will be given for each unit to assess students understanding of the topics. These will be similar to the homework exercises. This gives opportunity to remediate if the students are not demonstrating proficiency. Remediation can be individualized using online resources from the curriculum, including tutorials and practice questions with step-by-step solutions. Small group or whole class re-teaching may also be used depending on the needs of the students.

Unit Test: Each unit will culminate with a comprehensive unit test to assess the students' understanding of the unit as a whole. It will include exercises involving conceptual understanding, procedural fluency, and application and problem-solving skills.

Performance Tasks: Each unit test will include a multi-part, performance-based task that requires students to draw on their understanding of important math concepts and problem-solving skills.

Semester Final Exam: A comprehensive final exam will be given each semester to assess the students' retention of topics. Questions will address procedural fluency, application and problem-solving.